J Mater Sci (2009) 44:3026–3034
DOI 10.1007/s10853-009-3399-6

The random crack core model for predicting the longitudinal tensile strengths of unidirectional composites

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Received: 26 August 2008 / Accepted: 16 March 2009 / Published online: 2 April 2009

Abstract A perfect evolvement process of random crack cores is presented and a random crack core model for predicting the longitudinal tensile strengths of unidirectional composites is built in this paper. Based on the crack propagation rules, the numerical relationship of the number of random crack cores, evolvement probability of a random crack core evolving to critical size, and failure probability of a unidirectional composite are deduced. With considering some fibers breaks simultaneously and the influenced-length of the random crack cores increasing with the number of broken fibers, evolvement probability algorithms of a random crack core are developed based on the perfect evolvement process. At last, the longitudinal tensile strengths of unidirectional composites are predicted by the random crack core model, and the result shows that the random crack core model is more accurate than the classical theoretical models.

Introduction

Composites have been widely used in aerospace, military, and civil manufacturing industries, because of their high specific strength, specific modulus, wear and fatigue resistance [1]. However, the theoretical analysis on composite strengths is far behind. Such restriction has limited composites for further application. In despite of the fact that, a relatively integrated theory on composite micro-strength has been founded via micro-stress analysis combining with micro-strength criterions, it is still not satisfying due to the capability dispersing of constituent materials [2]. Statistical method, considering capability dispersing of constituent materials, has been used to analyze stochastic courses of composite failures. A virtue of this method is that it could transit analysis objects from micro-scale to macro-scale easily. In recent years, a number of statistical theories and models for predicting the longitudinal tensile strengths of unidirectional composites have been proposed.

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Using the weakest link theory, Gucer and Gurland [3] founded statistical chain of bundles model for predicting unidirectional composite strengths. Rosen [4] proposed the conception of “ineffective length” first and treated a unidirectional composite as a series of chains whose length is equal to the ineffective length. These are the well-known Gucer–Gurland–Rosen chain model based on the assumption that the failure of one chain means the invalidation of the whole composite. Nevertheless, the global load sharing principle adopted by the chain model is not deemed to be reasonable.

The fragmentation model was founded by Curtin [5, 6]. Based on the shear-lag theory, the density of breaks was described with considering load redistribution nearby fiber breakpoints. It was indicated that accretion of the break density is the direct reason of gradual stiffness decrease and final invalidation of composites. Although the density of breaks was analyzed in the model, congregative effect of breaks was ignored because of adopting the global load sharing principle.

A statistical theory of crack evolvement was proposed by Zweben, etc. [7, 8], in which the local load sharing principle was adopted for analyzing stress concentrative around cracks. The high local stress adds the break probabilities of intact fibers around cracks evidently. Even if the average stress level of composites is not enhanced, it is possible that intact fibers around cracks break on and on to the final invalidation of whole composites under the effect of local stress concentrations. The simplest way of the statistical crack evolvement theory assumed that a break of a discretional fiber means the invalidation of the whole composite [7]. The statistical crack evolvement theory was developed by Batdorf, etc. [9, 10]. It was considered that a composite failure will occur when a crack with some broken fibers is self-propagating, i.e. the crack evolves without any further applied stress.

A random critical-core model for predicting the longitudinal tensile strengths of unidirectional composites was proposed by Zeng [11]. Based on characters of crack propagations, it was pointed that the crack ineffective length increases with the number of broken fibers, which was deemed to be one of the main achievements in the model. However, unidirectional composites in the model were assumed impractically to be plane lamellas with two-dimensional fiber arrays.

Crack propagation is a complicated stochastic process. Based on recursive analysis, algorithm of crack evolvement probabilities for plane composites was proposed by Harlow [12]. Afterward the recursive algorithm was developed for unidirectional composites with fibers placed in hexagonal arrays by Pitt [13]. However, courses of the recursive analysis are complex, and tail-errors exist in the recursive analysis. Markov process without tail-errors was adopted to describe the course of crack evolvement by Goda [14], but the number of crack evolvement paths increases rapidly with the number of broken fibers. Therefore, Markov process is unwieldy for analyzing crack evolvement probabilities.

With the developments of computer technologies, Monte Carlo simulations were widely used for analyzing crack evolvements [15, 16]. However, these investigations were localized in small-scale composites. Difficulties still exist to simulate big-scale composite structures due to the limitation of current computing resources.

In this study, fibers are assumed to place in hexagonal arrays in unidirectional composites. On the basis of the crack propagation rules, the numerical relationship of the number of random crack cores, evolvement probability of a random crack core evolving to critical size and failure probability of a composite are deduced, and a theory of random crack cores is presented. To simplify the analysis, a perfect evolvement process of random crack cores was proposed, in which the local load sharing principle is adopted and the influenced-length of random crack cores increasing with the number of broken fibers is considered. The evolvement probability algorithm of random crack cores considering some fibers breaks simultaneously in evolvement processes of random crack cores is developed to increase the precision. Then, a random crack core model for predicting the longitudinal tensile strengths of unidirectional composites is built. At last, the random crack core model and its predictive results of unidirectional composite strengths are discussed by comparing with the referenced experimental and other theoretical results, and the size effect of composite strengths was analyzed by the random crack core model.

The random crack core theory

Preview of the random crack core theory

Due to strength dispersing of single fibers, some weak fibers may break first when a composite withstands a finite load, and then some random crack cores germinate in the composite. Intact fibers around random crack cores may break due to stress concentrations. When the size of a random crack core is big enough, the random crack core will be self-propagating, and then the whole composite will be invalidated ultimately.

Based on the crack propagation rules, a theory of random crack cores to predict the longitudinal tensile strengths of unidirectional composites is proposed. The primary contents of the random crack core theory are shown as follows: (1) the failure probability of a composite under a certain stress equals to the probability of existing a crack core evolving to critical size; (2) the number of random crack cores equals to the number of breaches germinated originally, and these crack cores distribute discretionarily and independently in the composite; (3) if the further evolvement probability of a random crack core is big enough, its further evolvement could be treated as an inevitable event, and the size of the random crack core is the critical size; (4) the event of a random crack core evolving to the critical size means the invalidation of the whole composite.

The number of random crack cores

Strengths of single fibers are dispersive, which was proved following tow-parameter Weibull distribution by Coleman [17]. The expression of strength distribution of single fibers is given as follows:

$$F(\sigma_f,\ \delta) = 1 - \exp\left[-\frac{\delta}{L_0}\left(\frac{\sigma_f}{\sigma_0}\right)^{\beta}\right] \quad (1)$$

where σ_f is the tensile stress of fibers, δ is the length of fiber, σ_0 is scaling parameter corresponding to fiber length L_0, β is shape parameter.

The number of random crack cores in a composite equals to the number of breaches germinated originally, and these crack cores distribute discretionarily and independently in the composite. If mechanical property of composites is unambiguous, the density of random crack cores, i.e., the number of random crack cores in a united length fiber, is determined only by the average tensile stress of fibers, moreover, increases with the average tensile stress. When the average tensile stress of fibers is σ_f, the density of random crack cores ρ is given as follows:

$$\rho = \frac{F(\sigma_f, \delta_0)}{\delta_0} \tag{2}$$

here δ_0 is the ineffective length of composites [4].

The ineffective length δ_0 of composites is determined jointly by characters of fibers, matrix and interface, which could be calculated by shear-lag model. With a hypothesis that tensile stress of broken fibers is linear with the distance from breakpoint in the stress-recovery zones, an approximate expression on the ineffective length δ_0 is given based on Kelly–Tyson shear-lag model [18] as follows:

$$\delta_0 = 2 \times \frac{\pi (d/2)^2 \sigma_f}{\pi d \tau_s} = \frac{d\sigma_f}{2\tau_s} \tag{3}$$

where d is the diameter of fibers; τ_s is the shear strength of fiber-matrix interface.

The number of random crack cores m equals to the product of the density of random crack cores and the total length of fibers V, which could be expressed as:

$$m = \mathrm{Int}\,[\rho V] = \mathrm{Int}\left[\frac{LN}{\delta_0} F(\sigma_f, \delta_0)\right] \tag{4}$$

where Int[ ] is the function of getting integers; L is the length of a composite; N is the total of fibers in the composite; the total length of fibers in the composite $V = LN$.

The critical size of random crack cores

If the further evolvement probability of a random crack core is big enough, the further evolvement could be treated as inevitable event, and the size of the random crack core at the moment is considered as the critical size. The criterion of the critical core size was given by Smith [19]. Considering fibers placed in hexagonal arrays, the critical size r of random crack cores requires to satisfy the following equations simultaneously:

$$F(K_{r-1}\sigma_f, \quad \delta_{r-1}) < 1 - e^{-1}$$

and

$$F(K_r \sigma_f, \quad \delta_r) \geq 1 - e^{-1} \tag{5}$$

where $K_i = \max(K_{i,j}),\ i < j \leq n_{con}$, and $K_{i,j}$ is the stress concentration factor of the fiber whose serial number is j when i fibers have broken in the random crack core; n_{con} is the biggest serial number of fibers enduring concentrative stresses when i fibers have broken in the random crack core; δ_i is the influenced-length of the random crack core with i broken fibers. Computational methods of these variables will be given in the section "Analysis on evolvement probabilities of random crack cores".

Failure probabilities of composites

In the random crack core theory, a number of random crack cores distribute discretionarily and independently in a composite, and the failure probability of the composite under a certain stress equals to the probability of existing a crack core evolving to critical size. Consequently, the failure probability of a composite can be expressed as:

$$G(\sigma_f) = 1 - [1 - P_r(\sigma_f)]^m \tag{6}$$

where $G(\sigma_f)$ is the failure probability of a composite under fiber stress σ_f; $P_r(\sigma_f)$ is the probability of a random crack core evolving to the critical size r, which will be worked out in the section "Analysis on evolvement probabilities of random crack cores".

Analysis on evolvement probabilities of random crack cores

The perfect evolvement process of random crack cores

Evolvement of a random crack core is an indeterminate process, and evolvement paths are various. The evolvement process can be expressed as in Fig. 1. According to the local load sharing principle in refs. [20], once a fiber is broken, six fibers around the broken fiber are all possible to break. Considering the spatial symmetry, there is only one evolvement path of the random crack core evolving from one broken fiber to two. Corresponding to the number of broken fibers in a random crack core evolving from 1 to 3, 4, 5 and 6, the number of evolvement paths are 3, 11, 80, and 822, respectively. Goda [14] used Markov process to describe evolvements of a random crack core. However,

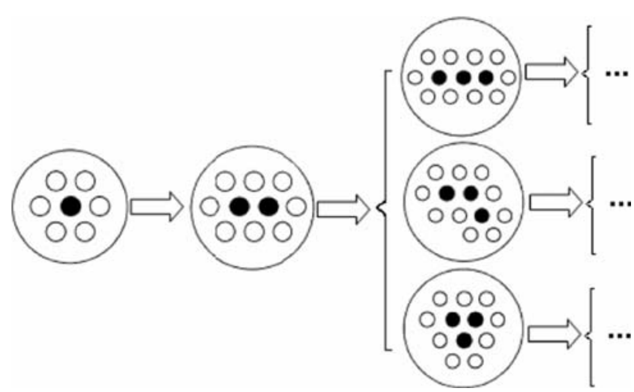

Fig. 1 The evolvement process of a random crack core

the number of evolvement paths increases rapidly with the number of broken fibers in a random crack core, and analysis on evolvement probabilities of a random crack core based on Markov process is rather complicated.

All-out paths are given in Markov process (without considering some fibers break simultaneously), but chance of every path makes a great difference with each other. The fiber, whose stress concentration factor is the largest, always has the biggest probability of break. For predigesting analysis and calculation, a perfect evolvement process of random crack cores was proposed, which is expected to replace the Markov process.

The perfect evolvement process of a random crack core was presented on the assumptions that the fiber whose stress concentration factor is the largest is the next broken fiber, and the failure probability of the fiber equals to the probability of any fiber around the random crack core breaking in a certain state. If stress concentration factors of some fibers are the coordinately largest, the nearest fiber from the center of the random crack core (the original crack core) is enacted to break next. If some of these fibers from the center of the random crack core are the coordinately nearest, the first fiber in counter-clockwise from the last broken fiber in these fibers is enacted to break next. According to aforementioned rules, fibers around a random crack core break in turn. Combining a load sharing principle, every evolvement step of a random crack core is unambiguous.

Combining the local load sharing principle in refs. [20], evolvement steps of a random crack core are determined. According to the sequence of failure fibers are marked in turn, shown in Fig. 2.

As a particular path of the Markov process, the perfect evolvement process of a random crack core is not only a path with the largest probability, but also a unique path insuring every evolvement step of the random crack core being a steady state. It is detected that if an evolvement step of a random crack core departs from corresponding steady state, the following steps will approach the steady states with much larger probabilities.

In fact, in an evolvement course of a random crack core, fibers do not always break one by one. In respect that the perfect evolvement process of a random crack core can be regard as a steady evolvement process, the failure sequence of fibers in Fig. 2 could also be used to analyze evolvement processes with some fibers breaking simultaneously.

Evolvement probabilities of random crack cores

Zeng [11] indicated that the influenced-length of a random crack core, i.e., the length of stress-recovery zones of broken fibers in the random crack core, increases with the number of broken fibers in the random crack core. The perfect evolvement process of random crack cores shows that crack cores always tend to be circular. However, the shear stress is not uniform along the crack boundary. Because the tensile stress of matrix is always changeless, the shear stress mainly distributes in zones near fibers around the random crack core. Taking example of a random crack core with three broken fibers, the main interfaces enduring shear stress are lined out strikingly in Fig. 3. A random crack core with i broken fibers is translated into an equivalent thick fiber whose section area is equal to the sum of i broken fibers. We consider that the perimeter of the equivalent thick fiber equals to the total length of main interfaces enduring shear stress approximately. An example is shown in Fig. 3.

The diameter of the equivalent thick fiber is expressed as:

$$D_i = 2\sqrt{\frac{i\pi(d/2)^2}{\pi}} = \sqrt{i}d \tag{7}$$

According to the Kelly–Tyson shear-lag model [18], the influenced-length of the random crack core can be expressed approximately as:

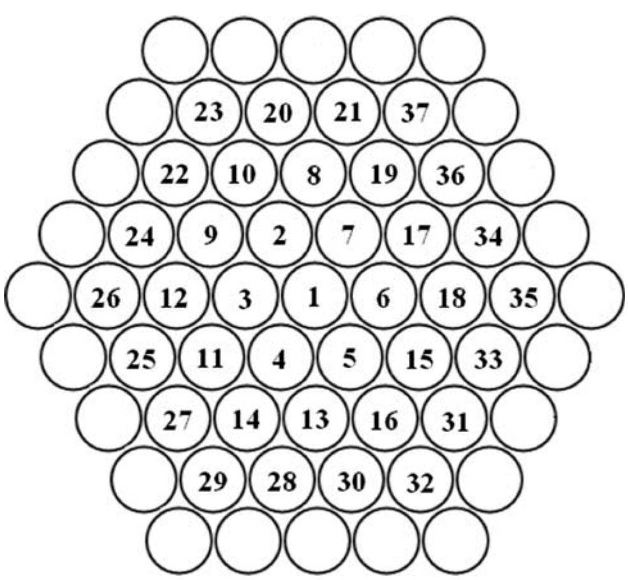

Fig. 2 The perfect evolvement process of a random crack core

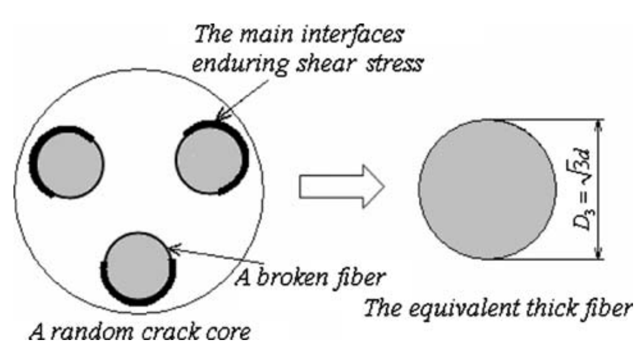

Fig. 3 Analysis on the main interfaces enduring shear stress

$$\delta_i \approx 2 \times \frac{n\pi(d/2)^2\sigma}{\pi D_i \tau_s} = \sqrt{i}\frac{d\sigma}{2\tau_s} = \sqrt{i}\delta_0 \tag{8}$$

when $i = 1$, $\delta_1 = \delta_0$.

When the number of broken fibers in a random crack core is i, the failure probability of the No. j fiber ($j > i$) under concentrative stress is shown as:

$$p_{i,j}(\sigma_f) = \frac{F(K_{i,j}\sigma_f\ ,\ \delta_i) - F(K_{i-1,j}\sigma_f\ ,\ \delta_{i-1})}{1 - F(K_{i-1,j}\sigma_f\ ,\ \delta_{i-1})} \tag{9}$$

here, $K_{i,j}$ is the stress concentration factor of the No. j fiber when the number of broken fibers in a random crack core is i, which could be calculated according to the local load sharing principle in refs. [20].

Based on the assumption of the perfect evolvement process of random crack cores, if the number of broken fibers in a random crack core is i, the probabilities of the number of broken fibers from i to $i + 1$, $i + 2$, and $i + 3$ in a step, marked, respectively, as $p_{i\to i+1}$, $p_{i\to i+2}$, and $p_{i\to i+3}$, are calculated as follows:

$$p_{i\to i+1}(\sigma_f) = \sum_{j=i+1}^{n_{con}} \frac{p_{i,j}}{1 - p_{i,j}} \prod_{j=i+1}^{n_{con}} (1 - p_{i,j}) \tag{10.1}$$

$$p_{i\to i+2}(\sigma_f) = \sum_{j=i+1}^{n_{con}-1} \sum_{k=j+1}^{n_{con}} \frac{p_{i,j}p_{i,k}}{(1 - p_{i,j})(1 - p_{i,k})} \prod_{j=i+1}^{n_{con}} (1 - p_{i,j}) \tag{10.2}$$

$$p_{i\to i+3}(\sigma_f) = \sum_{j=i+1}^{n_{con}-2} \sum_{k=j+1}^{n_{con}-1} \sum_{l=k+1}^{n_{con}} \frac{p_{i,j}p_{i,k}p_{i,l}}{(1 - p_{i,j})(1 - p_{i,k})(1 - p_{i,l})} \prod_{j=i+1}^{n_{con}} (1 - p_{i,j}) \tag{10.3}$$

where n_{con} is the biggest serial number of fibers enduring concentrative stresses at the moment. When the number of broken fibers in a random crack core is i, the probability of the number of broken fibers from i to $i + q$ in a step, marked as $p_{i\to i+q}$, can be deduced by analogy.

The evolvement probability of a random crack core with i broken fiber is shown as follows:

$$p_i(\sigma_f) = \sum_{q=1}^{n_{con}} p_{i\to i+q} = 1 - \prod_{j=i+1}^{n_{con}} (1 - p_{i,j}) \tag{11}$$

Chances of some fibers breaking simultaneously exist in evolvement processes of random crack cores, but it is unpractical to consider circumstantialities fully, because of their complexity. In previous investigations [12–14], the possibility of some fibers breaking simultaneously was always ignored. Because the perfect process changes megillah into simpleness, it becomes possible to consider some fibers breaks simultaneously. Here, multiform algorithms in many cases such as ignoring some fibers breaking simultaneously, considering two fibers breaking simultaneously and considering ω fibers breaking simultaneously, are discussed.

(1) Ignoring some fibers breaking simultaneously

When the stress level is low, the probability of some fibers breaking simultaneously is much smaller than that of fibers breaking one by one. For predigesting analysis, the possibility of some fibers breaking simultaneously is always ignored. Then, the evolvement possibility of a random crack core from one broken fiber to the critical size r is shown as follows:

$$P_r(\sigma_f) = \prod_{i=1}^{r-1} p_i \tag{12}$$

(2) Considering two fibers breaking simultaneously

The algorithm ignoring some fibers breaking simultaneously brings errors to calculated results consequentially. The algorithm considering some fibers breaking simultaneously is able to decrease errors. With considering two fibers breaking simultaneously, the evolvement possibility of a random crack core from one broken fiber to the critical size r could be calculated as follows:

$$P_1(\sigma_f) = 1 \tag{13.1}$$

$$P_2(\sigma_f) = p_{1\to 2} \tag{13.2}$$

$$P_i(\sigma_f) = p_{i-1\to i}P_{i-1} + (p_{i-2} - p_{i-2\to i-1})P_{i-2},\ 2 < i < r \tag{13.3}$$

$$P_r(\sigma_f) = p_{r-1}P_{n-1} + (p_{r-2} - p_{r-2\to r-1})P_{r-2} \tag{13.4}$$

(3) Considering ω fibers breaking simultaneously

With considering ω fibers breaking simultaneously, the evolvement possibility of a random crack core from one broken fiber to the critical size r could be calculated as follows:

$$P_1(\sigma_f) = 1 \tag{14.1}$$

$$P_i(\sigma_f) = \sum_{k=1}^{i-1} p_{k\to i}P_k,\ 2 \le i \le \omega \tag{14.2}$$

$$P_i(\sigma_f) = \sum_{k=1}^{\omega-1} p_{i-k\to i}P_{i-k} + (p_{i-\omega} - \sum_{k=1}^{\omega-1} p_{i-\omega\to i-k})P_{i-\omega}\ , \quad \omega < i < r \tag{14.3}$$

$$P_r(\sigma_f) = p_{r-1}P_{r-1} + \sum_{k=2}^{\omega} (p_{r-k} - \sum_{l=1}^{\omega-1} p_{r-k\to r-l})P_{r-k} \tag{14.4}$$

Discussion on the perfect evolvement process

The perfect evolvement process of random crack cores is a particular path of Markov process. It needs to be validated whether the perfect evolvement process reflects evolvement probability characteristics of random crack cores correctly. With the shape parameter $\beta = 5$, the possibilities of a random crack core evolving from one broken fiber to 2, 3, 4 and 5 are calculated through two methods based on the perfect process and Markov process, respectively. For comparing expediently, the crack influenced-length is treated as a constant, chances of some fibers breaking simultaneously are ignored, and expression (15) from refs. [14] is adopted to calculate one-step evolvement possibilities.

$$p_{i\to i+1}(\sigma) \approx \sum_{j=i+1}^{n_{con}} p_{i,j} \qquad (15)$$

Based on the perfect process and Markov process, respectively, possibility curves of the random crack core evolving from one broken fiber to 2, 3, 4 and 5 are plotted in Fig. 4, which are marked as P_2, P_3, P_4, and P_5, respectively. Obviously, the evolvement possibility curves of the random crack core calculated based on the perfect process and Markov process, respectively, are almost superposed. It is indicated that the perfect process is able to reflect evolvement probability characteristics of random crack cores commendably.

Based on the perfect process, the evolvement possibilities of a random crack core are calculated with considering the influenced-length of the random crack core increasing with the number of broken fibers. With the shape parameter $\beta = 5$, errors of the algorithm ignoring some fibers breaking simultaneously are analyzed by comparing to the algorithms considering some fibers breaking simultaneously.

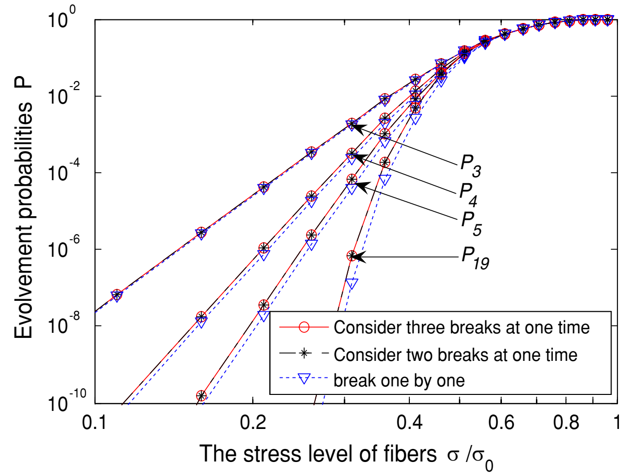

Fig. 5 Probability curves of random crack core evolvement

The evolvement probability curves of the random crack core are displayed in Fig. 5. P_i is the possibility of the random crack core evolving from one broken fiber to i, and the corresponding curves are derived from three algorithms such as ignoring some fibers breaking simultaneously, considering two fibers breaking simultaneously and considering three fibers breaking simultaneously. Compared with the calculated results with considering three fibers breaking simultaneously, it is shown that the algorithm ignoring some fibers breaking simultaneously makes the evolvement possibilities of the random crack core be underestimated and stress levels corresponding to a certain evolvement possibility be overrated. Nevertheless, the evolvement possibilities derived respectively from the algorithm considering two fibers breaking simultaneously and the algorithm considering three fibers breaking simultaneously are almost accordant (the errors of stress levels corresponding to a certain evolvement possibility are less than 0.3%).

Consequently, based on the perfect process, the algorithm considering two fibers breaking simultaneously is satisfactory for calculating evolvement possibilities of a random crack core.

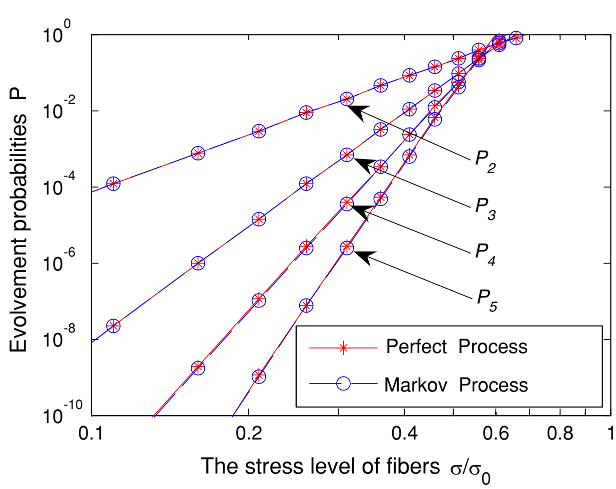

Fig. 4 Validation of the perfect process of random crack core evolvement

The random crack core model for predicting strengths

Failures of unidirectional composites are always controlled by fiber breaks and crack evolvements, therefore a random crack core model for predicting the longitudinal tensile strengths of unidirectional composites could be founded based on the theory and the perfect evolvement process of random crack cores. The statistical average longitudinal tensile strengths of unidirectional composites $\overline{\sigma_c}$ could be expressed as follows:

$$\overline{\sigma_c} = V_f\overline{\sigma_f} + (1 - V_f)\overline{\sigma_m} \quad (16)$$

where V_f is the volume fraction of fibers in the composite; $\overline{\sigma_f}$ is the average stress of fibers when the composite endures the maximal load; $\overline{\sigma_m}$ is the average stress of matrix when the composite endures the maximal load.

When the failure possibility of a composite is equal to 0.5, the stress level of fibers could be treated as the average stress of fibers $\overline{\sigma_f}$ corresponding to the composite enduring the maximal load approximately. It can be acquired by dichotomy with a stress zone $[a, b]$ determined first.

Figure 6 is the flow chart of the random crack core model for predicting the longitudinal tensile strengths of unidirectional composites. The parameters in the dark frames are known quantities. The distribution of fiber strengths could be investigated by tensile experiments of single fibers.

Validation and discussion

Predicting strengths of unidirectional resin matrix composites

Adopting the random crack core model, the longitudinal tensile strength of a unidirectional T300/5208 composite is predicted. Parameters of the composite are as follows [11]: the volume fraction of fibers $V_f = 0.7$; the diameter of fibers $d = 7$ μm; the tensile module of fibers $E_f = 230$; corresponding to the length of fiber $L_0 = 25$ mm, the scaling parameters $\sigma_0 = 2.98$ and the shape parameters $\beta = 7.68$; the tensile module of matrix $E_m = 3.45$ GPa; the shear strength $\tau_s = 25$ Mpa. Using the crack evolvement probability algorithm with considering two fibers breaking simultaneously and the change rule of the influence-length, the size effect of composite strengths is analyzed. The number of random crack cores m, the critical size r, and the average stress of fibers σ_f corresponding with a certain failure probability of a composite G are listed in Table 1.

Table 1 indicates that the average fiber stress σ_f corresponding with a certain failure probability G decreases with the composite size, namely, the average strength of the composite decreases with its size. It is disclosed that the degree of dispersing of composite strengths also decreases with the size. Obviously, the number of random crack cores increases with the composite size (i.e., the total length of fibers) and the average fiber stress σ_f. The critical size r of random crack cores decreases with the average fiber stress σ_f, so it increases with the composite size.

The test result of the longitudinal tensile strengths of the unidirectional T300/5208 composite is 1.50 Gpa [21]. According to contemporaneous test standard [22], the volume of the test segment of a unidirectional composite specimen is about 820–4,100 mm^3. Taking $V = 2{,}000$ mm^3, the longitudinal tensile strengths of the unidirectional T300/5208 composite is predicted by several classical theoretical models and the presented model. The predictive results and their errors are listed in Table 2.

As a result of adopting the global load sharing principle, the predictive values of Curtin's fragmentation model [6] are significantly higher than the experimental results, which are listed in Table 2. Zweben [7] treated a break of a discretional fiber as the criterion of composite invalidation in his statistical crack evolvement theory. So the strength of the composite is underestimated obviously. Batdorf [9] considered that composite failure will occur when a crack with some broken fibers is self-propagating. However, the influenced-length increasing with the number of broken fibers was not taken into account in Batdorf's model, which causes the crack evolvement probabilities be underestimated and the composite strength be overrated. Although the crack ineffective length increases with the number of broken fibers was taken into account in Zeng's model [11], the stress concentration factors around cracks are overrated and the composite strength is underestimated because of the impractical assumption of two-dimensional fiber arrays. Here, the present model of random crack cores considers the practical rule of the influence-length increasing with the

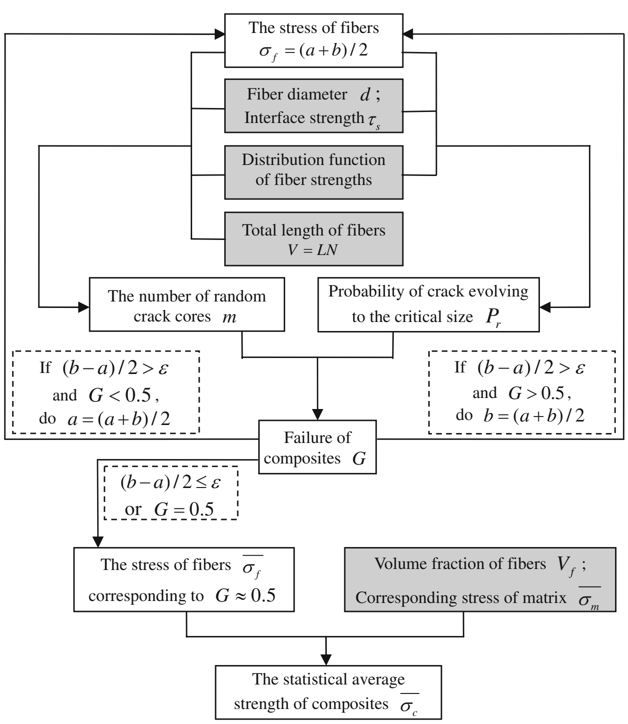

Fig. 6 The flow chart of the random crack core model for predicting the longitudinal tensile strengths of unidirectional composites

Table 1 Size effect of unidirectional T300/5208 composites

LN/m	$G = 0.1$			$G = 0.5$			$G = 0.9$		
	m	r	σ_f/GPa	m	r	σ_f/GPa	m	r	σ_f/GPa
0.1	2	5	2.8252	4	4	3.0271	6	3	3.2119
1	14	6	2.6032	22	5	2.7584	30	5	2.8821
10	88	6	2.4492	125	5	2.5634	165	5	2.6586
10^2	582	6	2.3188	817	6	2.4237	1029	6	2.4977
10^3	3933	6	2.2034	5410	6	2.2968	6681	6	2.3609
10^4	27033	6	2.0983	36709	6	2.1837	44831	6	2.2413
10^5	187640	6	2.0009	252809	6	2.0801	306639	6	2.1331
10^8	104678008	9	1.8544	128926416	9	1.9054	133797942	9	1.9147

Table 2 Theoretical results and their errors of the longitudinal tensile strength of unidirectional T300/5208 composites

	Curtin	Zweben	Batdorf	Zeng	Present
σ_c/GPa	2.50	0.44	1.87	1.35	1.49
Error/%	66.7	−70.1	24.7	−10.0	−0.7

number of broken fibers. Using the crack evolvement probability algorithm considering two fibers breaking simultaneously, the predictive results of the random crack core model are much more accurate than those of the classical theoretical models. Corresponding to the standard volume of the test segment 820–4,100 mm^3 [22], the predictive strength of the unidirectional T300/5208 composite is approximately 1.47–1.52 Gpa. Obviously, it is accordant with the test result.

Predicting strengths of unidirectional metal matrix composites

Nine panels of unidirectional SiC/Ti composites were made with fibers: SCS-6 and matrix: Ti-1100 by NASA Langley Research Center, which are marked A ∼ I in turn. The longitudinal tensile strengths of these composites and mechanical capabilities of fibers, matrix, interfaces were investigated experimentally by Gundel and Wawner [23]. The lengths of the composites are 38 mm, and the widths of them are 6.35 mm. The fiber volume fractions and the thicknesses of the composites are noted in Table 3. The diameter of fibers is 140 μm. Corresponding to the length of fiber $L_0 = 25.4$ mm, the scaling parameters σ_0 and the shape parameters β of Weibull function are shown in Table 3. Ti-1100 is elastic–plastic, and the matrix stresses $\overline{\sigma_m}$ corresponding to the composites enduring the maximal load are 935 Mpa approximately. The interface strengths are listed in Table 3.

Table 3 Correlative parameters of unidirectional SiC/Ti composites [23]

Panel	V_f	$h/\mu m$	σ_0/MPa	β	τ_s/MPa
A	0.15	1650	2950	9.1	200
B	0.15	1650	3930	10.1	200
C	0.18	1555	4310	13.9	200
D	0.20	1350	2890	5.8	200
E	0.20	1350	3850	12.3	200
F	0.26	1150	4270	12.3	75
G	0.28	1065	4640	12.6	75
H	0.30	910	3330	6.8	75
I	0.35	835	4410	11.6	75

The local load sharing principle [20] was deduced on the assumption that matrix did not participate in redistribution of the load. Normally, metal matrix have yielded before composites fail [23], so the local load sharing principle [20] and the random crack core model are also applicable to metal matrix composites. The random crack core model is adopted for predicting the longitudinal tensile strengths of the unidirectional SiC/Ti composites, and the predictive results and errors are noted in Table 4.

In Table 4, the predictive results of the longitudinal tensile strengths of the unidirectional SiC/Ti composites are accordant with the experimental results [23] basically, and the errors are receivable. It shows that the random crack core model is also accurate for predicting metal matrix composites.

Conclusions

The random crack core model for predicting the longitudinal tensile strengths of unidirectional composites is built in this study. The model, based on the random crack core theory and the perfect evolvement process in this study, is

Table 4 Comparison of predictive results of the random crack core model with experimental results of the longitudinal tensile strength of unidirectional SiC/Ti composites

Material	Experiment σ_c [23] (MPa)	Predictive results	
		σ_c (MPa)	Error (%)
A	1150	1158	0.7
B	1250	1269	1.5
C	1360	1397	2.7
D	1230	1227	−0.2
E	1380	1373	−0.5
F	1496	1543	3.1
G	1724	1670	−3.1
H	1327	1390	4.7
I	1716	1783	3.9

validated by comparing the predictive results and measurements of unidirectional composites. Profiting from the assumption of fibers placed in hexagonal arrays, adopting the local load sharing principle, and considering the influenced-length of random crack cores increasing with the number of broken fibers, the random crack core model is much better than the classical theoretical models.

The perfect evolvement process of random crack cores is proposed and validated, which predigests analysis and calculation of crack evolvement probabilities remarkably. Based on the perfect evolvement process, errors of the algorithm ignoring some fibers breaking simultaneously are discussed. The results show that the algorithm, ignoring some fibers breaking simultaneously, makes the crack evolvement possibilities be underestimated and corresponding stress levels be overrated, while the algorithm considering two fibers breaking simultaneously is generally accurate enough for predicting composite strengths.

The size effect of composite strengths is analyzed and incarnated by the random crack model. In the model, the number of random crack cores and their critical size increase with the composite size, while the average strengths of composites and the degree of its dispersing decrease with it.

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